
SEMINAR
on
COMPLEX AND HYPERCOMPLEX ANALYSIS

Online, Departamento de Matemática

26/05/2021 & 02/06/2021, 16:00

Spin representations, Fourier transform, and symbol calculus

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In this seminar, we consider representation theory to develop a full symbol calculus of pseudo-differential operators on the group $\text{Spin}(4)$ in the sense of Ruzhansky-Turunen-Wirth. The essential tools are the $\text{Spin}(4)$ -representations, its matrix coefficients, and the Fourier transform on $\text{Spin}(4)$, which is a matrix-valued operator. To give a more comprehensive view of the topic and the research area we divide the seminar into two parts. In the first part, we give an overview of Spin representations and how to construct models of Spin representations in the Clifford algebra setting based on the work of F. Sommen and his co-authors. Then we concentrate on the study of $\text{Spin}(3)$ representations, its connection with $\text{SU}(2)$, and S^3 representations, being S^3 identified with the unit sphere in the quaternions. We show recurrence relations for the matrix coefficients, establish a differential calculus for left/right invariant differential operators, and a symbol calculus for $\text{Spin}(3)$. In the second part of the seminar, we construct the $\text{Spin}(4)$ representations in the spaces of simplicial harmonic polynomials and simplicial spinor-valued monogenic polynomials decomposing them as the tensor product of $\text{Spin}(3)$ -representations. Using the Kronecker product and the properties of $\text{Spin}(3)$ representations we study recurrence relations for the matrix coefficients of $\text{Spin}(4)$ representations and establish a differential and symbol calculus for some left/right invariant differential operators. With the Fourier transform on $\text{Spin}(4)$ in hand and a family of admissible first-order difference operators chosen we study pseudo-differential operators on the group $\text{Spin}(4)$. We obtain results concerning the ellipticity and the global hypoellipticity of pseudo-differential operators in $\text{Spin}(4)$, in terms of their matrix-valued full symbols. Several examples of first and second-order globally hypoelliptic differential operators are given, in particular, of operators that are locally not invertible nor hypoelliptic but globally are.

Zoom Link:

<https://videoconf-colibri.zoom.us/j/83947134957?pwd=THVJbDkxV3lHQ1drNGNOajhnLzladz09>

Meeting ID: 839 4713 4957

Password: 958454

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