SEMINAR

on

COMPLEX AND HYPERCOMPLEX ANALYSIS

Sala Sousa Pinto, Departamento de Matemática

03/12/2021, 16:15

From Hermite to Zernike. Orthogonal Polynomials on the unit disk. Applications in Optics

Teresa E. Pérez
Instituto de Matemáticas (IMAG) and Departamento de Matemática Aplicada
Universidad de Granada (Espanha)

In 1865, Charles Hermite [1] published a paper (divided into four parts) introducing bivariate orthogonal polynomials on the disk to solve an bivariate approximation problem proposed by P. Chebyshev. Despite the fact that a priori the problem seems to be a simple generalization of standard orthogonal polynomials to the bivariate case, the solution presents several obstacles. C. Hermite then introduced the concept of biorthogonality in this context and orthogonal polynomials systems on the disk were described explicitly.

Zernike polynomials were introduced by Frits Zernike en 1934 [2] to describe the wavefront in the formation of images. In 2000, the Optical Society of America (OSA) adopted them as standard patron in Optics and Ophthalmology. Mathematically, Zernike polynomials are polynomials in two variables orthogonal on the unit disk, and are represented in polar coordinates as a product of a radial part (a univariate Jacobi polynomial) and an angular part represented by spherical harmonics.

In this talk we describe the families of bivariate orthogonal polynomials on the disk introduced by C. Hermite, show that Zernike polynomials are a particular case of disk polynomials, and we analyse the main applications of Zernike polynomial in Optics.

References:

[1] C. Hermite, *Sur quelques développement en séries des fonctions*, Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences. Tome Soixantième. Janvier - Juin 1865. Paris. 370-377, 432-440, 461-466, 512-518.

[2] F. Zernike, *Beugungstheorie des Schneidenverfahrens und Seiner Verbesserten Form, der Phasenkontrastmethode*, Physica. 1 (1934), 689-704.