Introduction to complex analytic sets

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A complex analytic set is a subset in a complex manifold that is locally defined by a finite number of holomorphic equations.

The theory of complex analytic sets is situated on crossing of many domains of mathematics, first of all, analysis of several complex variables, algebraic geometry, and it is widely used in many other domains, e.g., dynamical systems. The fact that the preimage of an analytic subset under a holomorphic map is analytic follows immediately from definition. One of the main results of the theory is Remmert Proper Mapping Theorem stating that the image of analytic subset under a proper holomorphic map is analytic.

The classical Chow Theorem states that every analytic subset of complex projective space is algebraic: zero locus of a finite collection of homogeneous polynomials.

The local analytic set theory studies germs of analytic sets and is closely related to singularity theory.

In this introductory talk we will discuss the above-mentioned results and present a proof of one of the first results of the theory: the Preparatory Weierstrass Theorem stating that under a "non-degeneracy condition", a germ of holomorphic function in n+1 variables is a product of a non-zero holomorphic germ (unity in the local ring) and a polynomial in one of the variables.

Afterwards we concentrate ourselves on germs of analytic sets in two dimensions, introduce Newton diagram of germ of holomorphic function and show how the geometry of its zero locus can be partially described by its Newton diagram.

This will be a talk for non-specialists, just familiar with the classical analysis of one complex variable.