

SEMINAR

Grupo de Análise Funcional e Aplicações Functional Analysis and Applications Group

On the direct problem for Laguerre-Hahn orthogonal polynomials on the real line

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Abstract

Orthogonal polynomials on the real line, $P_n(x) = x^n + \dots$, $n \geq 0$, are fully characterized through the three-term recurrence relation $P_{n+1}(x) = (x - \beta_n)P_n(x) - \gamma_n P_{n-1}(x)$, $n = 0, 1, 2, \dots$, with initial conditions $P_{-1}(x) = 0$, $P_0(x) = 1$. The numbers β_n, γ_n are commonly called the recurrence coefficients.

In this talk we shall focus on the families of orthogonal polynomials on the real line related to Stieltjes functions, S , that satisfy a Riccati differential equation

$$AS' = BS^2 + CS + D, \quad (1)$$

where A, B, C, D are polynomials. These are known as Laguerre-Hahn orthogonal polynomials (LHOP). Note the special cases: the so-called classical orthogonal polynomials - the Hermite, Laguerre, Jacobi polynomials, when $B \equiv 0$ and A, C such that $\deg(A) \leq 2, \deg(C) = 1$; the semi-classical orthogonal polynomials, when $B \equiv 0$ and $\deg(C) > 1$.

The main goal is to study the so-called direct problem for LHOP: to deduce properties of the recurrence coefficients of the orthogonal polynomials from the knowledge of the polynomials A, B, C, D involved in the Riccati equation (1). For some families of LHOP, it is shown that the recurrence coefficients satisfy some forms of discrete Painlevé equations.

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