# SEMINAR

## Grupo de Análise Funcional e Aplicações
## Functional Analysis and Applications Group

## Jones factorization and Rubio de Francia extrapolation for matrix weights

### David V. Cruz-Uribe, OFS
The University of Alabama

## Abstract

In the 1990s, Nazarov, Treil and Volberg introduced a generalization of the scalar Muckenhoupt $A_p$ condition to matrix weights. Let $W$ be a $d \times d$ symmetric, positive definite matrix weight function. For $1 < p < \infty$, we say $W$ is in matrix $A_p$ if $[W]_{A_p} = \sup_Q \fint_Q \left( \fint_Q |W^{1/p}(x)W^{-1/p}(y)|_{op}^{p'} \, dy \right)^{p/p'} dx < \infty$, where the supremum is taken over all cubes in $\mathbb{R}^n$. These weights arise in the study of stationary processes, operator theory on vector-valued functions, and systems of PDEs. They showed that the Hilbert transform is bounded on $L^p(W)$, the space of vector-valued functions with norm $\|f\|_{L^p(W)} = \left( \int_{\mathbb{R}^n} |W^{1/p}(x)f(x)|^p \, dx \right)^{1/p}$. Later, Christ and Goldberg extended this result to all Calderón-Zygmund singular integrals, and a theory of two-weight norm inequalities was introduced by DCU, Isralowitz and Moen.

In the 1990s, Nazarov, Treil and Volberg posed two related problems: extend the Jones factorization theorem and the Rubio de Francia theory of extrapolation, to matrix $A_p$ weights. Since 2011, an important open question has been to prove the $A_2$ conjecture for matrix weights: it is conjectured that the sharp exponent on the $A_p$ characteristic for matrix weights is the same as for scalar weights, $\max\{1, \frac{1}{p-1}\}$.

In this talk we will discuss the solution of the first two problems. Joint with Marcin Bownik, we have proved exact generalizations of factorization and the sharp constant extrapolation theorem that was used to prove the scalar $A_2$ conjecture. The proofs required the development of a new family of tools for working with convex-set valued functions. This let us generalize the ideas in the convex-body sparse domination theorem of Nazarov, Petermichl, Treil and Volberg. We defined the analog of the Hardy-Littlewood maximal operator on convex-set valued functions, and proved weighted norm inequalities for this operator on $L^p(W)$ (suitably extended to convex-set valued functions). This allowed us to define a Rubio de Francia iteration algorithm, the fundamental tool in the proof of both factorization and extrapolation.

### Room *Sousa Pinto*
### February 15, 2024 - 14:30