Departamento de Matemática
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CIDMA CENTRO DE I&D EM MATEMÁTICA E APLICAÇÕES
CENTER FOR R&D IN MATHEMATICS AND APPLICATIONS

Seminário do Grupo de Álgebra e Geometria

Automorphisms and Isomorphisms of Maps in Linear Time

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Resumo

A map is a 2-cell decomposition of a closed compact surface, i.e., an embedding of a graph such that every face is homeomorphic to an open disc. An automorphism of a map can be thought of as a permutation of the vertices which preserves the vertex-edge-face incidences in the embedding. Every automorphism of a map determines an angle-preserving homeomorphism of the surface. While it is conjectured that there is no "truly subquadratic" algorithm for testing map isomorphism for unconstrained genus, we present a linear-time algorithm for computing the generators of the automorphism group of a map on an orientable surface of genus g, parametrized by the genus g. A map on an orientable surface is uniform if the cyclic vector of sizes of faces incident to a vertex v does not depend on the choice of v. The algorithm applies a sequence of local reductions and produces a uniform map, while preserving the automorphism group. In order not to enlarge the automorphism group we introduce labels on darts of the maps. The labels are modified when an elementary reduction is applied. If g>1, then the reduction terminates in a finite family of uniform maps. It follows that one can solve the problem for these maps applying brute force. The spherical and toroidal case require more sophisticated approach. We have modified and generalized an original algorithm by Hopcroft and Wong (1974) to solve the problem. The automorphism group of the original map can be reconstructed from the automorphism group of the associated uniform map in linear time. We also extend the algorithm to non-orientable surfaces by making use of the antipodal double-cover. The algorithm can be used to solve the map isomorphism problem between maps (orientable or non-orientable) of bounded negative Euler characteristic.

The results were obtained in collaboration with Ken Ichi Kawarabayashi, Bojan Mohar and Peter Zeman, and they were published in ACM Trans. Alg. 2024.

Sala Sousa Pinto, 25 de setembro de 2024, 14:00

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