

# SEMINAR

## Grupo de Análise Funcional e Aplicações Functional Analysis and Applications Group

### A $p$ -Laplacian problem in $\mathbb{R}^N$ with singular, convective, critical reaction

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#### Abstract

The talk is devoted to the problem

$$\begin{cases} -\Delta_p u = \lambda w(x) f(u, \nabla u) + u^{p^*-1} & \text{in } \mathbb{R}^N, \\ u > 0 & \text{in } \mathbb{R}^N, \\ u(x) \rightarrow 0 & \text{as } |x| \rightarrow +\infty, \end{cases}$$

where  $N \geq 2$ ,  $1 < p < N$ , and  $\lambda > 0$ . The nonlinear term  $f : (0, +\infty) \times \mathbb{R}^N \rightarrow (0, +\infty)$  is a continuous function which is singular in the first variable and  $p$ -sublinear with respect to the second one. The weight  $w : \mathbb{R}^N \rightarrow (0, +\infty)$  satisfies suitable summability and decay conditions. The problem exhibits several features:

- the perturbation  $f$  is singular, i.e., it blows up when the solution vanishes;
- $f$  encompasses also convection terms, that is, depending on the gradient of the solution;
- the 'dominating' reaction term has critical growth;
- the setting is the whole  $\mathbb{R}^N$ ;
- pointwise decay (at infinity) of the solutions is required.

We will present an existence result that combines variational methods, truncation techniques, and concentration compactness arguments, together with set-valued analysis and fixed point theory. In addition, De Giorgi's technique, a priori gradient estimates, and nonlinear regularity theory will be employed to ensure local  $C^{1,\alpha}$  regularity of solutions, as well as their pointwise decay at infinity. The result is new even in the non-singular case, also for the Laplacian. Joint work with Laura Baldelli.

**Room Sousa Pinto**  
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