

Math Colloquium CIDMA/UAlg

An object-wise approach to Categorical Algebra

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Abstract:

How do we distinguish “good” objects in a setting with weak algebraic properties ... without using elements?

Many topics in mathematics involve the study of “objects” X endowed with certain properties, and “arrows” $f: X \rightarrow Y$ which preserve them. For example, homomorphisms between groups, linear maps between vector spaces, continuous maps between topological spaces. With that data we can consider new constructions or properties that can be, conveniently, represented by commutative diagrams of arrows. Diagrams are the essence of Category Theory.

By handling diagrams, Category Theory provides an element-free approach to many element-based mathematical fields, such as general algebraic ones. An element-free approach has the advantage of giving a better perception of whatever properties are being studied, as well as their possible generalisation to other settings.

Distinguishing “good” objects in weaker settings is a common practice. This led us to develop in [1] an object-wise approach to some important notions occurring in Categorical Algebra. The main aim of that work was to provide a categorical-algebraic characterisation of groups amongst monoids and of rings amongst semirings; of course, no elements involved.

The goal of this talk is to give an idea of how we may use a categorical approach to answer the initial question.

[1] A. Montoli, D. Rodelo, and T. Van der Linden, *Two characterisations of groups amongst monoids*, J. Pure Appl. Algebra **222** (2018) 747–777.
