UNIVERSITY OF AVEIRO
Department of Mathematics

Seminar of the

Complex and Hypercomplex Analysis Group (GACH)

`Wednesday, 27 May 2026, 15h`

`Room 11.2.23, Departamento de Matemática`

Amílcar Branquinho

Universidade de Coimbra, Portugal

Introduction to the theory of orthogonal polynomials, Markov chains, and integrable systems

The theory of orthogonal polynomials can be divided into two main branches: algebraic theory and analytic theory. These two branches share a great deal in common, and the dividing line between them is often thin. It could be said that this reflects the broader relationship in mathematics between Algebra and Analysis. One facet relates to the algebraic dimension of the theory, which maintains intimate connections with special functions, combinatorics, and algebra. All discrete polynomials, as well as their classical analogues, are situated within this framework. Furthermore, much of the current state of research regarding multivariate orthogonal polynomials aligns closely with this algebraic component of the theory.

The other part is the analytical aspect of the theory. Its methods are analytical and deal with typical questions in analysis, or questions that have arisen and are related to other parts of mathematical analysis. General properties occupy a smaller part of the analytical theory, and the majority is divided into two main and extremely rich branches: orthogonal polynomials on the real axis and on the unit circle. The connection between orthogonal polynomials and other branches of mathematics is truly impressive. Without being exhaustive, we can mention continued fractions, operator theory (Jacobi operators), moment problems, analytic functions, interpolation, Padé approximation, quadrature, approximation theory, numerical analysis, electrostatic problems, statistical quantum mechanics, special functions, number theory (irrationality and transcendence), combinatorics, random matrices, birth-and-death processes, data classification and compression, the Radon transform, and computed tomography.

In this presentation, we will provide an introduction to the theory of orthogonal polynomials from a *naive* perspective, aiming to illustrate various mathematical disciplines associated with this field. We will examine the spectral theorem for Jacobi-type operators and explore its applications within Toda systems.

The GACH seminar is supported by CIDMA under the Portuguese Foundation for Science and Technology (FCT), Grants UID/04106/2025 and UID/PRR/04106/2025.