UNIVERSITY OF AVEIRO
Department of Mathematics

Complex and Hypercomplex Analysis

SEMINAR
on
COMPLEX AND HYPERCOMPLEX ANALYSIS

Sala Sousa Pinto (2^o piso), Departamento de Matemática

07/10/2017, 14:00

An existence theory for nonlinear equations on metric graphs

Matthias Hofmann
Universidade de Lisboa, Lisboa, Portugal

The purpose of this talk is to develop a general existence theory for constrained minimization problems for functionals defined on function spaces on metric measure spaces (M, d, μ). We apply this theory to functionals defined on metric graphs G, in particular L^2-constrained minimization problems of the form

$$E(u) = \frac{1}{2}a(u,u) - \frac{1}{q}\int_K |u|^q dx,$$

where $q > 2$ and $a(\cdot,\cdot)$ is a suitable symmetric, sesquilinear form on some function space on G and $K \subset G$ is given. We show how the existence of solutions can be obtained via decomposition methods using spectral properties of the operator A associated with the form $a(\cdot,\cdot)$ and discuss the spectral quantities involved. An example that we consider is the higher-order variant of the stationary NLS (nonlinear Schrödinger) energy functional with potential $V \in L^2 + L^\infty(G)$

$$E^{(k)}(u) = \frac{1}{2}\int_G |u(k)|^2 + V(x)|u|^2 dx - \frac{1}{q}\int_K |u|^q dx,$$

defined on a class of higher-order Sobolev spaces $H^k(G)$ that we introduce. When K is a bounded subgraph, one has localized nonlinearities, which we treat as a special case. When $k = 1$ we also consider metric graphs with infinite edge set as well as magnetic potentials.Then the operator A associated to the linear form is a Schrödinger operator, and in the L^2-subcritical case $2 < q < 6$, we obtain generalizations of existence results for the NLS functional as for instance obtained by Adami, Serra and Tilli [JFA 271 (2016), 201-223], and Cacciapuoti, Finco and Noja [Nonlinearity 30 (2017), 3271-3303], among others.

This seminar is supported by CIDMA - Center for Research and Development in Mathematics and Applications, and FCT - Fundação para a Ciência e a Tecnologia, within project UID/MAT/04106/2019.

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