asymptotics of orthogonal polynomials for a weight with a jump on [-1,+1]

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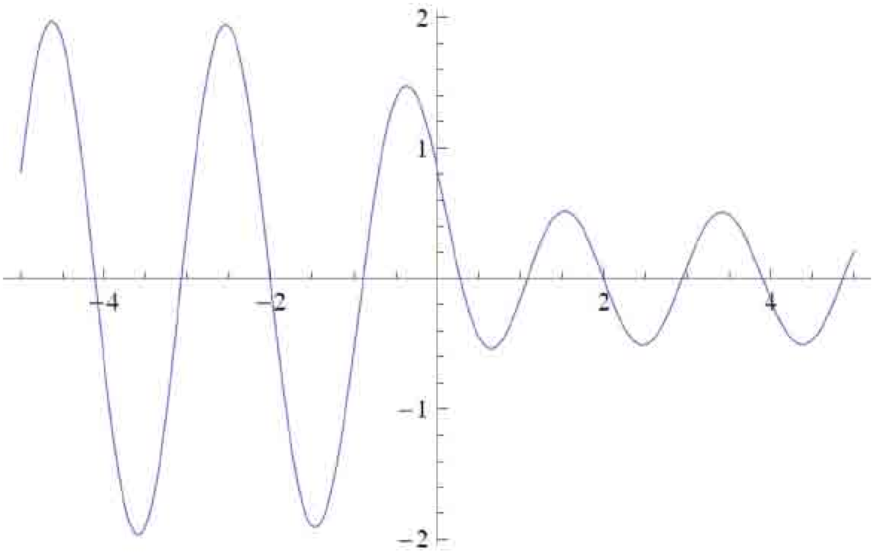

Szegö is the founder of the modern general theory of orthogonal polynomials. In particular, he was the first to prove the crucial asymptotic result for polynomials orthogonal on the interval with respect to weights that satisfy a condition that today bears his name. In the case of the classical Jacobi weights, we can derive this asymptotics (both on and away from the interval of orthogonality, as well as at its endpoints) using several identities that these orthogonal polynomials satisfy: the differential equation, the Rodrigues formula, integral representation, etc.
However, in a general situation, the problem is much more difficult. Starting from the 80's, many new asymptotic results were found for various classes of weights, and the breakthrough was partially motivated by the development of the tools from potential theory and operator theory.
An important new technique for obtaining asymptotics for orthogonal polynomials in all regions of the complex plane is based on the characterization of the orthogonal polynomials by means of a Riemann–Hilbert problem for 2 _ 2 matrix-valued functions due to Fokas, Its, and Kitaev, combined with the steepest descent method of Deift and Zhou.
In a recent paper by Kuijlaars and collaborators, the complete asymptotic expansion for the orthogonal polynomials with respect to a Jacobi weight modified by a real analytic and strictly positive function was obtained. It showed that this modification does not affect essentially the local behavior of the polynomials, which has a direct implication to the study of the so-called universality property and the "clock behavior" (uniform spacing) of the zeros.
A very different situation arises when the weight has a singularity on the interval of orthogonality. The case of a zero was analyzed in the work of Vanlessen, but the case of the jump singularity remained open.
We consider polynomials that are orthogonal on a finite interval [_1, 1] with respect to a modified Jacobi weight with a jump at the origin. From our analysis we obtain strong uniform asymptotics of the monic orthogonal polynomials in the whole complex plane, as well as the first terms of the asymptotic expansion of the main parameters (leading coefficients of the orthonormal polynomials and the recurrence coefficients). In particular, we prove a conjecture of A. Magnus regarding the asymptotics of the recurrence coefficients.
The main focus is on the local analysis at the origin that is made using confluent hypergeometric functions. We study the asymptotics of the Christoffel–Darboux kernel in a neighborhood of the jump, and this is the first explicit example of a non-sine reproducing kernel of a de Branges space that arises as a universality limit in the bulk of a fixed measure of orthogonality.
We also show that the zeros of the orthogonal polynomials no longer exhibit the clock behavior.