GREEN'S THEOREM FOR GENERALIZED FRACTIONAL DERIVATIVES

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In 1828, the English mathematician George Green (1793-1841), who up to his forties was working as a baker and a miller, published an essay where he introduced a formula connecting the line integral around a simple closed curve with a double integral. Within years, this result turned out to be useful in many fields of mathematics, physics and engineering. Generalizations of Green's theorem have chosen different directions, and are known as the Kelvin-Stokes and the Gauss-Ostrogradsky theorems. In our work we proved Green's theorem for generalized partial fractional (noninteger order) derivatives.

Fractional Calculus (FC) is a generalization of the standard (integer order) differential calculus, in the sense that it deals with derivatives of real or complex order. FC was born on 30th September 1695. On that day, L'Hopital wrote a letter to Leibniz, where he asked about Leibniz's notation of nth order derivatives. L'Hopital wanted to know the result for the derivative of order n = 1/2. Leibniz replied that "one day, useful consequences will be drawn" and, in fact, his vision became a reality. The study of noninteger order derivatives rapidly became a very attractive subject. Many different forms of fractional derivative operators were introduced, e.g., Hadamard, Riemann–Liouville, Caputo, Riesz, Cresson, Katugampola, Klimek, and many results of classical analysis were extended to the noninteger case. Unfortunately, since fractional integrals and derivatives can be defined in many different ways, in each case all results must be considered separately. In our work we introduced more general operators that, by choosing appropriately special kernels, reduce to the standard fractional integrals and derivatives found in the literature. Using the new operators, we generalized Green's theorem[1].

This work received The Grunvald-Letnikov award for the best student paper – theory, on the Inter-national Conference "Fractional Differentiation and Applications", that took place in Nanjing, China, from 14th to 17th May 2012 (see Fig. 1).

FIGURE 1

Grunvald-Letnikov award, May 2012