inversion of the noisy Radon transform on SO(3) by Gabor frames and sparse recovery principles

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One of the modern methods in determining the structure of polycristalline materials is the so-called X-ray diffraction tomography. For each incidence ray one measures the diffraction pattern and from this information one desires to determine the crystallographic structure of the material. Mathematically this is done by inverting the so-called spherical Radon transform which is an ill-posed inverse problem. In the paper “Inversion of the noisy Radon transform on SO(3) by Gabor frames and sparse recovery principles”, Appl. Comput. Harmon. Anal. (2011) a new method for obtaining a stable approximation of the inverse of the spherical Radon transform was established. X-ray tomography of crystallographic structures using diffraction experiments is a computationally expensive task. To give an idea of the complexity of the problem just by measuring as few as 100 incidence rays and 100 scattered rays one obtains already 10 000 measurements. The developed numerical method reduces the problem greatly by constructing new building blocks (so-called spherical Gabor frames) which allow us to use sparse recovery principles (only a few building blocks have non-zero coefficients) while maintaining a stable approximation of the inverse of the spherical Radon transform. The proposed approach is composed by basic building blocks of the coorbit theory on homogeneous spaces, Gabor frame constructions and variational principles for sparse recovery. The performance of the finally obtained iterative approximation is studied through several experiments and it was shown that this new method works well with noisy data.