UNIVERSITY OF AVEIRO
Department of Mathematics

Complex and Hypercomplex Analysis

SEMINAR

on

COMPLEX AND HYPERCOMPLEX ANALYSIS

Sala Sousa Pinto, Departamento de Matemática
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Computing fractional powers of operators by quadrature rules

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We consider the numerical approximation of $\mathcal{L}^{-\alpha}, \alpha \in (0,1)$. Here $\mathcal{L}$ is a self-adjoint positive operator acting in an Hilbert space $\mathcal{H}$ in which the eigenfunctions of $\mathcal{L}$ form an orthonormal basis of $\mathcal{H}$, so that $\mathcal{L}^{-\alpha}$ can be written through the spectral decomposition of $\mathcal{L}$. In other words, for a given $g \in \mathcal{H}$, we have

$$\mathcal{L}^{-\alpha} g = \sum_{j=1}^{+\infty} \mu_j^{-\alpha} \langle g, \varphi_j \rangle \varphi_j$$

where μ_j and φ_j are the eigenvalues and the eigenfunctions of $\mathcal{L}$, respectively, and $\langle \cdot, \cdot \rangle$ denotes the $\mathcal{H}$-inner product. This problem finds immediate application when solving equations involving a fractional diffusion term like $(-\Delta)^\alpha$, where Δ denotes the standard Laplacian, and this is the main reason for which in recent years a lot of attention has been placed on the efficient approximation of fractional powers.

Starting from the integral representation

$$\mathcal{L}^{-\alpha} = \frac{\sin(\alpha\pi)}{\pi} \int_0^{+\infty} s^{-\alpha} (s\mathcal{I} + s\mathcal{L})^{-1} ds, \qquad \alpha \in (0,1),$$

where $\mathcal{I}$ is the identity operator in $\mathcal{H}$, after suitable changes of variables and quadrature rules one typically finds rational approximations of the type

$$\mathcal{L}^{-\alpha} \approx \mathcal{R}_{n-1,n}(\mathcal{L}), \quad \mathcal{R}_{n-1,n}(\lambda) = \frac{p_{n-1}(\lambda)}{q_n(\lambda)}, \quad p_{n-1} \in \Pi_{n-1},\, q_n \in \Pi_n,$$

where n is equal or closely related to the number of points of the quadrature formula. In this seminar we present a comparative analysis of the most reliable existing methods based on quadrature rules, with particular attention to the error estimate and the asymptotic rate of convergence. The analysis is given in the infinite dimensional setting, so that all results can be directly applied to the discrete case, independently of the discretization used.

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