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Γ-CONVERGENCE AND SHELL EQUATIONS

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Abstract

We consider bending of elastic "isotropic" media governed by Láme equations. The boundary conditions are classical Dirichlet-Neumann mixed type. The domain $\Omega^h := -\mathcal{C} \times (-h, h)$ is of thickness $2h$. Here $\mathcal{C} \subset \mathcal{S}$ is a smooth subsurface of a closed hypersurface $\mathcal{S}$ with smooth nonempty boundary $\partial\mathcal{C}$.

The object of the investigation is what happens with the above mentioned mixed boundary value problems when the thickness of the layer converges to zero $h \to 0$. It is shown that the corresponding BVPs converge in the sense of Γ-convergence to a certain BVPs on the mid surface $\mathcal{C}$: The BVP for the Láme equation converges to the BVP for the shell equation on the mid surface.

The suggested approach is based on the representation of the Láme operator in terms of Günter's tangential and normal (to the surface) derivatives. Namely, if ν is the unit normal vector field on the surface, extended in the domain Ω_h, the Günter's derivatives read $\mathcal{D}_j := \partial_j - \nu_j \mathcal{D}_4,\ \mathcal{D}_4 = \partial_\nu = \sum_{k=1}^{3} \nu_k \partial_k,\ j = 1, \ldots, n$ and the Láme operator in the domain $\mathcal{L}_{\Omega^h} = -\mu\,\Delta - (\lambda + \mu)\,\nabla \mathrm{div}$ is represented as follows: $\Delta_{\Omega^h} = \sum_{j=1}^{4} \mathcal{D}_j^2 + 2\mathcal{H}_\mathcal{C}\mathcal{D}_4, -\ \mathcal{L}_{\Omega^h} = -\mu\,\Delta_{\Omega^h} - (\lambda + \mu)\,\nabla_{\Omega^h} \mathrm{div}_{\Omega^h}$. Here $\mathcal{H}_\mathcal{C}$ is the mean curvature of the surface $\mathcal{C}$ and $\Delta_{\Omega^h} = \partial_1^2 + \partial_2^2 + \partial_3^2$,

$$\nabla_{\Omega^h}\varphi := \Big\{\mathcal{D}_1\varphi, ..., \mathcal{D}_4\varphi\Big\}^\top,\ \mathrm{div}_{\Omega^h}\mathbf{U} := \sum_{j=1}^{4} \mathcal{D}_j U_j^0 + \mathcal{H}_\mathcal{C} U_4^0,$$

$\mathbf{U} = (U_1, U_2, U_3)^\top,\ U_j^0 := U_j - U_4^0,\ U_4^0 := \langle \nu, \mathbf{U}\rangle,\quad j = 1, 2, 3$ are the laplace operator, gradient and divergence represented in Günter's derivatives, $\langle \nu, \mathbf{U}\rangle$ denotes the Cartesian scalar product.

The work is carried out in collaboration with T. Buchukuri (Tbilisi).

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