

SEMINAR

Grupo de Análise Funcional e Aplicações Functional Analysis and Applications Group

Certain Extensions of a Result of Siegel

Pedro Ribeiro

Department of Mathematics, Faculty of Sciences of University of Porto

Abstract

Let $k \in \mathbb{N}$ and consider the zeta function attached to the sum of k squares,

$$\zeta_k(s) = \sum_{n_1, \dots, n_k \in \mathbb{Z}^k \setminus \{0\}} \frac{1}{(n_1^2 + \dots + n_k^2)^s}, \quad \operatorname{Re}(s) > \frac{k}{2}. \quad (1)$$

Siegel [2] proved very sharp results about the distributions of zeros of $\zeta_k(s)$, for $k \geq 12$. His results include an estimate for the number of zeros of $\zeta_k(s)$ in the line of symmetry $\operatorname{Re}(s) = \frac{k}{4}$. Following the work of Dixit, Kumar, Maji and Zaharescu [1], in our presentation we will discuss how one can study zeros of combinations of the form

$$F_k(s) = \sum_{j=1}^{\infty} c_j \pi^{-s-i\lambda_j} \Gamma(s+i\lambda_j) \zeta_k(s+i\lambda_j) \mathcal{F}(s+i\lambda_j; z), \quad \operatorname{Re}(s) = \frac{k}{4},$$

where $\mathcal{F}(s+i\lambda_j; z)$ is a function connected to Kummer's function. We will also show how to approach this study when k is any positive real number.

This talk is based on joint work with Semyon Yakubovich.

References

- [1] A. Dixit, R. Kumar, B. Maji and A. Zaharescu, Zeros of combinations of the Riemann Ξ -function and the confluent hypergeometric function on bounded vertical shifts, *J. Math. Anal. Appl.* **466** (2018), 307–323.
- [2] C. L. Siegel, Contributions to the Theory of Dirichlet L-series and the Epstein zeta-functions, *Ann. of Math.*, **44** (1943), 143–172.

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