Group of Algebra and Geometry

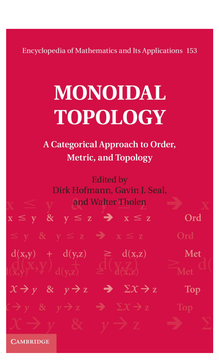
Group description

The group of Algebra and Geometry brings together researchers with interest in Geometry, Algebra, Combinatorics, Topology, Logic and Category Theory. The group has also a strong interest in various aspects of education of Mathematics, in particular, it develops the interdisciplinary project Geometrix for the implementation of different adaptive computer aided learning environments for the teaching of Mathematics.

Some Achievements

In collaboration with members of other groups of CIDMA, we have ongoing research in Coding Theory. In particular, we were able to construct convolutional codes that attain the maximum possible distance for some fixed parameters of the code, namely, the rate and the Forney indices. These results answer some famous open questions on distances and constructions of convolutional codes. We also studied the construction of superregular matrices over small finite fields, and created a new cryptosystem based on convolutional codes and are studying its security properties.

Together with researchers from Portugal, Canada, Belgium and Switzerland, we have developed a general framework for topology based on enriched category theory.



The basic general theory together with numerous new results ranging from topology and approach theory to domain theory were published in 2014 in the book *Monoidal Topology* at Cambridge University Press.

Besides the development of the general framework, over the past years we have particularly contributed to the investigation of function space structures, completeness properties, and duality theory. Our work combining duality theory with quantale-enriched categories was recently published in *Advances in Mathematics*.

We established a novel research field lying in the intersection of Mathematics and Computer Science. This was achieved as a result of systematic development of new modal logics (hybrid and dynamic) suitable to describe the static component of a computational system. Semantics, proof calculi, notions of bisimilarity and refinement, and characterisation results, were systematically built, providing a foundation for the engineering of reconfigurable systems and the development of computational tools for (semi-) automatic verification of logic requirements. This research, a collaboration with colleagues from HASLab – INESC TEC (University of Minho), is reported in several scientific papers in important international journals. Our results were developed in the context of three successive, competitively funded research projects, one post-doctoral and 4 PhD grants supported by FCT. In the context of this work, in 2013, Alexandre Madeira was awarded with the *IBM scientific prize* for his PhD thesis.

Among our achievements in geometry we mention the classification of all regular polytopes of rank $n-3$ ($n \geq 9$) and $n-4$ ($n \geq 11$) whose automorphism group has degree n; as well as proving that maximal rank for a regular polytope with automorphism group being an alternating group of degree n is $\lfloor (n-1)/2 \rfloor$ ($n \geq 12$). These results were recently published in *Transactions of the American Mathematical Society* and *Proceedings of The London Mathematical Society*.

Thematic lines

Since its beginning, the interdisciplinary project GEOMETRIX is an important part of our group. It integrates a multidisciplinary team, bringing together mathematicians, computer programmers, educators and graphic designers around a common goal, the study, creation, development and use of inclusive teaching and learning environments, crisscrossing all levels of education, guaranteeing their access to vulnerable groups such as children with special needs and the elderly. Geometrix relies in research on mathematics, technology and mathematics education for the development of (computational) tools to support and empower students with knowledge and skills in mathematics, from basic to higher education. It is also committed to the creation of extension activities where mathematics / culture / community / university are articulated in a natural and meaningful way.

The thematic line MATEAS also includes a project relevant for our group, SIACUA, with an interactive Web application for autonomous learning in higher education, having already more than five thousand instances of parameterized questions with detailed solutions, among other learning objects like short videos, providing feedback computed by a Bayesian user model, and aiming to use other artificial intelligence tools in computer systems for learning mathematics.

International Collaboration

We have close collaborations with other centers in our fields in Portugal (in particular at the Universities of Porto, Minho and Coimbra) and abroad (Toronto, Brussels, Milano, Louvain, Capetown, Salamanca, among others).

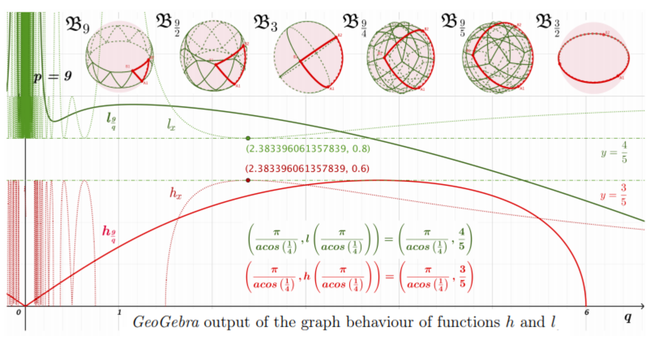
Our group enjoys regularly visits of well-known researchers in our fields, among them Patrick Blackburn (Roskilde), Peter Cameron (St. Andrews), Dimitri Leeman (Brussels), Walter Tholen (Toronto), Maria Manzano (Salamanca), Rolf Hennicker (Munich), Martín Escardó (Birmingham), Michael Giudici (Perth), Asia Weiss (Toronto).

We were also invited to visit established research centers such as the Fields Institute, University of Auckland, Université Catholique de Louvain and Università degli Studi di Milano.

Current Research Topics

Central research topics of our group include

- Algebraic Combinatorics and Discrete Geometry,
- Category theory and its applications to Algebra and Topology,
- Algebraization of logics,
- Coding theory and Cryptography,
- Bayesian student models for mathematics learning,
- Interactive educational software,
- Combinatorial and geometrical properties of shperical tilings.



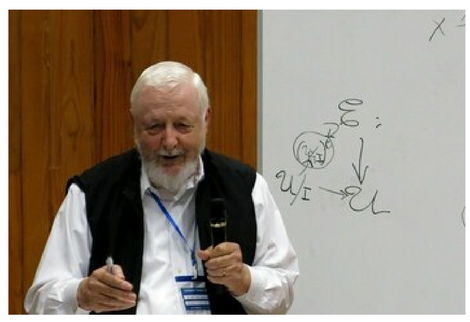
Visibility and Recognition

As a sign of international recognition, we received over the past years several invitations for invited main talks at conferences, among them are the principal conference in category theory in 2017 (CT2017 – International Category Theory Conference, Vancouver) and the 37th Linz Seminar on Fuzzy Set Theory. We also belonged to several thesis committees in and outside Portugal (Coimbra, Lisbon, Toronto, Lausanne). Finally, we belong to Editorial boards in principal journals of our fields (Theory and Applications of Categories, Acta Universitatis Matthiae Belii).

In 2015 we organized the principal annual conference in Category Theory CT 2015, which counted with 124 participants from 24 countries,

`http://sweet.ua.pt/dirk/ct2015`

celebrating the 50th anniversary of international category theory meetings and featured a special lecture by Bill Lawvere: *Alexander Grothendieck and the modern conception of Space.*



Bill Lawvere during his talk at CT2015

Members of our group also participated in the organization of the conference All Kinds of Mathematics Remind me of You celebrating the 70th Anniversary of Peter Cameron July 2017 in Lisbon. Peter Cameron was also a long-term visitor of our group.

Elisa Fernandes, Dimitri Leemans and Peter Cameron in Aveiro in 2015

We participate in various funded national and international projects, among them

- Hybrid Intensional Logic,
- Logical Tools for Systems Biology,
- Categorical Methods in Non Abelian Algebra.

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