Compressed Sensing for Quaternionic Signals

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A long lasting dream is the possibility to reconstruct signals when only few information is available. The main difficulty here is the fact that classic methods do not apply anymore in general and other methods work with only a high probability. In the last decade a new paradigm for this problem emerged, so-called Compressed Sensing. The related theory has currently a wide range of applications in many fields of *Computational and Applied Mathematics*. In the paper Compressed Sensing for Quaternionic Signals by S. Hartmann, N. Gomes, and U. Kaehler, published in *Complex Analysis and Operator Theory* in 2017, the authors show that this paradigm also works in non-commutative structures. Here, one major drawback is the fact that Linear Algebra of matrices over non-commutative structures is a tricky business. For instance, in the case of quaternionic matrices the question of left eigenvalues makes no sense while the problem of right eigenvalues is a non-linear problem. In the paper the authors demonstrated that there are classes of quaternionic matrices for which compressed sensing can be applied with a high probability while giving explicit estimates of this probability. As a direct application Compressed Sensing for quaternionic matrices arising in color representation of images are studied and implemented. Examples are given where the knowledge of only 15% or 25% of image data is enough for full reconstruction. This opens the way to applications of compressed sensing to more complicated quaternionic representations like of images where not only the position is codified but also directions (movements) as well as in other fields.