CIDMA CENTRO DE I&D EM MATEMÁTICA E APLICAÇÕES
CENTER FOR R&D IN MATHEMATICS AND APPLICATIONS

universidade de aveiro
Departamento de Matemática

(OGTC group)

Wavelets: basic constructions and applications

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Segunda-feira e terça-feira, dias 20 e 21 de jan./2020, das 14h00 s 18h00, Sala Sousa Pinto.

A short preface

This is an introductory mini-course for graduate and PhD students. We begin with basic notions of signal processing such as Fourier series, orthonormal bases and the fast Fourier transform. We discuss several applications, where we see disadvantages of standard orthonormal bases and motivate their generalizations: Riesz bases, biorthogonal systems and frames. Then we consider in detail the first wavelets: Haar system an its “dual” Shannon-Kotelnikov system, passing to more advanced Battle-Lemaire and Meyer systems, and finally, to compactly-supported wavelets, including Daubechies wavelets. The general construction of wavelets is introduced by the fundamental concept of multiresolution analysis, which motivates refinement functional equations as starting point. This allows us to derive the properties of compactly-supported wavelets, such as approximations, local and global regularity and relation to fractals. Finally we discuss applications to signal processing and numerical methods in PDE.

Program of the course

1. Fourier series and Fourier transform. Time and frequency domains in continuous and discrete time. The fast Fourier transform. Some applications of orthonormal systems (signal and image processing, numerical methods in PDE). Motivation for wavelets.

2. Haar systems on a segment and on a line. Regularity, approximation properties and applications of the Haar system. Shannon-Kotelnikov wavelets. The sampling theorem.

3. The general construction of wavelets. Multiresolution analyis. Scaling (refinable) function as the “mother” of a wavelet system. Fast computing of wavelet coefficients. The cascade algorithm. Examples. Time-frequency localization. Uncertainty constant. Wavelets of Battle-Lemaire. Wavelets of Meyer.

4. Advantages of compact support. Do compactly-supported smooth wavelets exist ? Grippenberg's theorem. Refinement equations. Mask and symbol. Existence, uniqueness and formula for the solution.

5. Compactly-supported functions. Linear independence, stability, and orthogonality of integer shifts. Cohen's criterion and the criterion in terms of cycles and symmetric roots. Approximations by shifts of a compactly-supported function. Strang-fix conditions. Sum rules.

6. Daubechies wavelets. Construction and main properties.

7. Three methods to estimate regularity of compactly-supported wavelets. The joint spectral radius. Wavelets and fractals: what is in common ?

Literature

[1] I.Daubechies, *Ten lectures on wavelets*, SIAM, Philadelphia (1992).

[2] I.Y.Novikov, V.Yu.Protasov, and M.A.Skopina, *Wavelets theory*, AMS, Translations Mathematical Monographs, 239 (2011).

[3] M.W.Frazier, *Introduction to wavelets through linear algebra*, Springer-Verlag, New-York (1999).

[4] P.A.Wojtaszczyk, *Mathematical introduction to wavelets*, London Math. Soc., Stunent texts, 37 (1997).

[5] B.S.Kashin and A.A.Sahakian, *Orthogonal series*, AMS, Providence (1989).

[6] O.Christensen, *Frames and bases. An introductory course*, Series: Applied and Numerical Harmonic Analysis, Birkhauser Basel (2008).

[7] V.Yu.Protasov, *Fractal curves and wavelets*, Izvestiya Mathematics, 70 (2006), No 5, 975-1013.

[8] L.Villemoes, *Wavelet analysis of refinement equations,* SIAM J. Math. Anal. 25 (1994), No 5, 1433-1460.

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